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A SIMPLE MODEL OF RADIATION HEAT TRANSFER FROM A CLOUD OF BURNING PARTICLES IN A CONFINED GAS STREAM

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Abstract—The paper describes a simple model for thermal radiation, based on the Milne–Eddington approximation, which has been used with some measure of success to predict heat-transfer rates in an axially-symmetrical pulverized-fuel flame.

NOMENCLATURE

a,	absorption coefficient;
D,	particle diameter;
G, H, K,	radiation fluxes;
I	intensity of outward radiation;
i,	intensity of radiation in direction j;
J	intensity of inward radiation;
j_r, j_θ, j_z,	length components in cylindrical-polar co-ordinates;
n,	number of particles per unit volume and diameter interval;
q,	radiation flux;
r,	radius;
s,	scattering coefficient;
T,	temperature of radiating medium;
T_W,	temperature of combustion-chamber wall;
z,	axial distance;
γ, ϕ,	angles defined in Fig. 1;
ε,	emissivity of particles;
ε_W,	wall emissivity;
θ,	azimuth angle in cylindrical polar co-ordinates;
σ,	Stefan–Boltzmann constant:
Ω,	solid angle.

INTRODUCTION

THE TWO-DIMENSIONAL recirculating flow of a reacting gas mixture is described by a set of elliptic partial differential equations which express the conservation of chemical species, momentum and energy. When the reacting medium is capable of absorbing, emitting and scattering thermal radiation it is necessary to consider additional equations, which are usually written in integro-differential form. Heat transfer from a pulverized-fuel flame is predominantly by radiation which can only be predicted by solving these two coupled sets of equations, a task presenting formidable mathematical problems if attempted in its entirety.

The pre-requisite for calculating radiation heat transfer is a knowledge of the physical state and optical properties of the radiating medium throughout the flow field. Until recently, calculation methods were limited by the difficulty of obtaining this information to the consideration of highly-simplified models of the real flows. An opportunity to improve on this situation has been afforded by the development [1, 2] of numerical methods for predicting recirculating flows with heat and mass transfer. These methods involve the use of suitable physical hypotheses for the mechanism of turbulent diffusion together with a finite-difference solution scheme for the conservation equations. Problems introduced by considering a two-phase reacting mixture can be overcome by postulating general similarity between the turbulent diffusion of gas and solid-fuel particles. In some respects the radiation problem is simplified by the presence of burning particles: the optical properties of the flame can be deduced by assuming the particles to be "grey" and the gas to be transparent to radiation.

The particular flow considered here is that of a pulverized-coal flame confined in a cylindrical furnace [3]. The complete calculations, which were carried out in parallel with an extensive experimental programme, have been reported in some detail elsewhere [4]; time and space limitations did not then allow details of the radiation model to be given. It is to remedy this deficiency that the present account has been written.

ANALYSIS

The transfer equation of thermal radiation

Steady-state radiation in an emitting, absorbing and scattering medium is governed by the integro-differential equation:

$$(\hat{j}\,.\,\nabla)\,i = -\,(a+s)\,i + \frac{a\sigma T^4}{\pi} + \frac{s}{4\pi}\int_0^{4\pi} i\,\mathrm{d}\Omega \qquad (1)$$

in which the individual terms have the following meanings:

1. $(\hat{j}\,.\,\nabla)\,i$ is the rate of change of intensity i with respect to length in the direction $\boldsymbol{j}$
2. the first term on the right-hand side represents the attenuation of i due to scattering and absorption in the medium
3. $a\sigma T^4/\pi$ is the amount by which i is augmented by emission
4. the integral term represents the amount by which i is augmented due to scattering of all incident beams of energy into the $\boldsymbol{j}$ direction.

The derivation of equation (1) is given in [5] together with a number of approximate methods of solution. Of these the Milne–Eddington method may be used to reduce equation (1) to two ordinary differential equations. Consider a ray of intensity i at the point $P(r,\ \theta,\ z)$, as shown in Fig. 1, and let ϕ be the angle made by

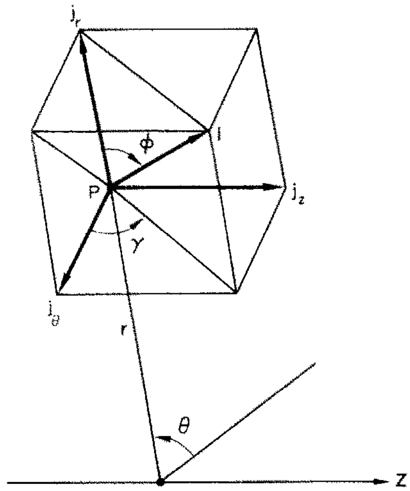

FIG. 1. Resolution of radiant intensity i into r, θ, z components.

the ray at P with the radial co-ordinate. Ordinary differential equations are formed by:

1. integrating equation (1) over all solid angles
2. multiplying both sides of equation (1) by $\cos\,\phi$ and again integrating over all solid angles.

In order to perform these integrations, however, the variation of i with ϕ and γ must be known or assumed. The basis of the Milne–Eddington approximation is to assume uniformity of intensity in the inward and outward directions; that is, i is supposed independent of γ and:

$$\left.\begin{aligned} i(\phi) &= I \quad \text{for } -\pi/2 < \phi \leqslant \pi/2 \\ i(\phi) &= J \quad \text{for } \quad \pi/2 < \phi \leqslant 3\pi/2 \end{aligned}\right\} \qquad (2)$$

Integration of the transfer equation

The first term of equation (1) may be written:

$$(\hat{j}\,.\,\nabla)\, i = j_r\left(\frac{\partial i}{\partial r}\right)_{\theta, z} + \frac{j_\theta}{r}\left(\frac{\partial i}{\partial \theta}\right)_{r, z} + j_z\left(\frac{\partial i}{\partial z}\right)_{r, \theta} \quad (3)$$

where j_r, h_θ and j_z are the scaler components of the unit vector j which are shown in Fig. 1 and whose values are:

$$\left.\begin{aligned} j_r &= \cos\phi \\ j_\phi &= \sin\phi\cos\gamma \\ j_z &= \sin\phi\sin\gamma \end{aligned}\right\} \quad (4)$$

When the system possesses axial symmetry:

$$\left[\left(\frac{\partial i}{\partial \theta}\right)_{\phi, \gamma}\right]_r = 0. \quad (5)$$

A further simplification can be obtained by assuming that the energy exchange by radiation is predominantly in the radial direction, so that:

$$\left(\frac{\partial i}{\partial z}\right)_{r, \theta} \to 0. \quad (6)$$

It may be noted, however, that the other components of $(\partial i/\partial \theta)$ are not necessarily zero. In addition, consideration of the system geometry produces the relationships:

$$\left.\begin{aligned} \left(\frac{\partial \phi}{\partial \theta}\right)_{r, z} &= -\cos\gamma \\ \left(\frac{\partial \gamma}{\partial \theta}\right)_{r, z} &= \frac{\sin\gamma}{\tan\phi} \end{aligned}\right\} \quad (7)$$

Equation (3) can now be reduced, by means of these relationships, to:

$$\begin{aligned} (\hat{j}\,.\,\nabla)\, i &= \cos\phi\left(\frac{\partial i}{\partial r}\right)_{\theta, z} \\ &\quad - \frac{1}{r}\sin\phi\cos^2\gamma\left(\frac{\partial i}{\partial \phi}\right)_{\gamma, r, z} \\ &\quad + \frac{1}{2r}\cos\phi\sin 2\gamma\left(\frac{\partial i}{\partial \gamma}\right)_{\phi, r, z}. \end{aligned} \quad (8)$$

Integration over all solid angles produces;

$$\begin{aligned} \int_0^{4\pi} (\hat{j}\,.\,\nabla)\, i\, \mathrm{d}\Omega &= \frac{\mathrm{d}}{\mathrm{d}r}\int_0^{2\pi}\int_0^{\pi} i\cos\phi\sin\phi\,\mathrm{d}\phi\,\mathrm{d}\gamma \\ &\quad - \frac{1}{r}\int_0^{2\pi}\cos^2\gamma\int_0^{\pi}\sin^2\phi\left(\frac{\partial i}{\partial \phi}\right)\mathrm{d}\phi\,\mathrm{d}\gamma \\ &\quad + \frac{1}{2r}\int_0^{\pi}\cos\phi\sin\phi\int_0^{2\pi}\sin 2\gamma\left(\frac{\partial i}{\partial \gamma}\right)\mathrm{d}\gamma\,\mathrm{d}\phi. \end{aligned} \quad (9)$$

The final term can be discarded by virtue of the "profile" assumption that i is independent of of γ. Integration by parts then yields:

$$\int_0^{4\pi} (\hat{j}\,.\,\nabla)\, i\, \mathrm{d}\Omega = \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(qr) \quad (10)$$

in which q is the flux of radiant energy defined by:

$$q = 2\pi\int_0^{\pi} i\cos\phi\sin\phi\,\mathrm{d}\phi. \quad (11)$$

This concludes the integration of the left-hand side of equation (1); that of the right-hand side is quite straightforward and the final integrated equation is:

$$\frac{1}{r}\,.\,\frac{\mathrm{d}}{\mathrm{d}r}(qr) = a(4\sigma T^4 - G) \quad (12)$$

in which:

$$G \equiv \int_0^{4\pi} i\,\mathrm{d}\Omega. \quad (13)$$

A second ordinary differential equation is obtained by multiplying equation (1) throughout by $\cos\phi$ and integrating in a similar way to get:

$$\frac{\mathrm{d}H}{\mathrm{d}r} + \frac{H}{r} - \frac{K}{r} = -(a + s)\, q \quad (14)$$

where:

$$H \equiv -\frac{2\pi}{3}\int_0^{\pi} i\,\frac{\mathrm{d}}{\mathrm{d}\phi}(\cos^3\phi)\,\mathrm{d}\phi \quad (15)$$

$$K \equiv \pi \int_0^{\pi} i \sin^3 \phi \, d\phi. \tag{16}$$

G, H and K are connected as follows through the intensity profile defined by equation (2):

$$G = 2\pi(I + J) \tag{17}$$

$$H = G/3 \tag{18}$$

$$K = G/3 \tag{19}$$

while, from equation (11):

$$q = \pi(I - J). \tag{20}$$

Thus the transfer equation can be written as two ordinary differential equations for the variables (qr) and H:

$$\frac{1}{r} \cdot \frac{d}{dr}(qr) = a(4\sigma T^4 - 3H) \tag{21}$$

$$\frac{dH}{dr} = -\frac{a+s}{r}(qr). \tag{22}$$

Boundary conditions

On the centreline of a cylindrical combustion chamber axial symmetry requires:

$$q = 0. \tag{23}$$

The proportion of incident radiation transmitted through the furnace wall is small and the opportunity has been taken to simplify the boundary condition there by making the convenient, but not essential, assumption that the wall is opaque. The sum of the absorptivity and reflectivity is then equal to unity and the wall condition is:

$$\mu J = \varepsilon_W \sigma T_W^4 + (1 - \varepsilon_W)\mu I \tag{24}$$

where ε_W and T_W are respectively the emissivity and temperature of the wall. With the aid of equations (17) and (18) this condition is conveniently expressed as:

$$H = \frac{4}{3}\sigma T_W{}^4 + \frac{4}{3}q\left(\frac{1}{\varepsilon_W} - \frac{1}{2}\right). \tag{25}$$

Optical properties of the particle cloud

Consider a control volume in a cloud of grey diffusely radiating particles in which all emission, absorption and scattering is supposed due solely to the presence of particles, the carrier gas being wholly transparent to radiation. The control volume has unit area normal to the ray of incident radiation and thickness dj along its length. If there are n particles per unit cloud volume and diameter interval dD, the area fraction blocked to radiation is

$$\frac{\pi}{4} dj \int_0^{\infty} nD^2 \, dD.$$

Thus an absorption coefficient, defined as the proportion of energy absorbed per unit length, is given by:

$$a = \frac{\pi\varepsilon}{4} \int_0^{\infty} nD^{2'} \, dD. \tag{26}$$

Similar reasoning produces the scattering coefficient:

$$s = \frac{\pi}{4}(1 - \varepsilon) \int_0^{\infty} nD^2 \, dD. \tag{27}$$

Since it is assumed that only the particles emit radiation, the temperature appearing in equation (21) must be some appropriate average of the surface temperatures of the particles in the control volume.

CALCULATIONS

Equations (21) and (22) are conveniently solved, for the prescribed boundary and flow conditions, by the numerical procedure outlined in the Appendix. Figure 2 shows results typical of calculations which have been made for comparison with experimental data. Complete details of the calculations are given in [4]; it need only be noted here that an extended version of the Pun–Spalding procedures [1, 2] was used to solve the conservation equations for chemical species, momentum and enthalpy which are coupled to the radiation equations by the appearance, as a "source" of enthalpy, of the term div $\bar{q}$.

The particles, in five size groups, were assumed to have the same diffusive transport properties as the reacting gases and to burn at a finite rate. In these calculations the particle-surface temperature was set at 200 °C above the local gas temperature, and the combustion-chamber walls to be cooled to 400°K. The gas flow was strongly recirculating.

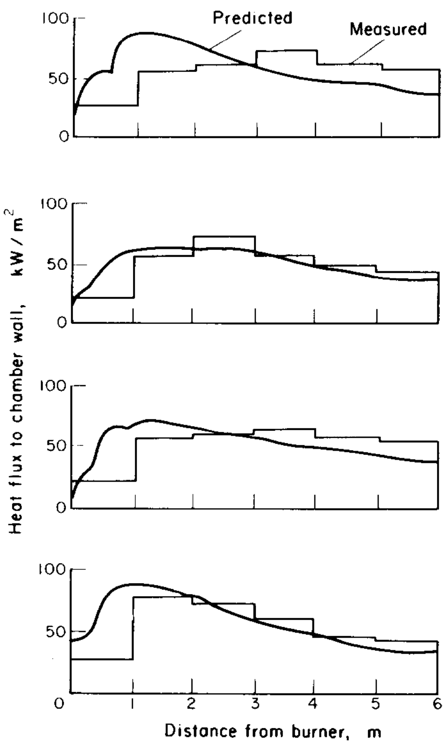

FIG. 2. Comparison of measured and calculated values of the heat flux to the walls of the BCURA p.f. combustion chamber [4].

The computed heat fluxes to the chamber walls, shown in Fig. 2 for four cases of non-swirling flow, are seen to be very fair agreement with measured values, particularly at distances greater than 1 m from the burner. Although the complete calculations, including those for the recirculating gas flow, involve a considerable number of approximations, it is possible to attribute this discrepancy mainly to one particular simplification. As noted above, particle burning is very simply treated; the particles are assumed instantaneously to attain temperatures directly related to the local gas temperature. No provision is made for a period of particle heating while infinitely fast reactions for the volatile elements in the fuel near the burner lead to high local gas temperatures. One obvious consequence of this assumption, which will be improved in future calculations, is that the rate of emission is overpredicted. Further downstream it is to be expected that particle and gas temperatures will be closer in magnitude and, consequently, better agreement is obtained with the data.

CONCLUSION

A simple model for thermal radiation, based on a Milne–Eddington approximation for axially symmetrical geometries, has been used to predict heat transfer processes in a pulverized-fuel flame. In spite of the apparent simplicity of the model, and the assumptions made in computing other flow phenomena not discussed in this paper, the predictions made are considered remarkably good. There is, however, considerable scope for improvement both in respect of the physical assumptions and of details in the numerical procedures; the chief of these are detailed in [4]. In fact, those improvements envisaged in the short-to-medium term are connected with aspects of the flow other than radiation heat transfer. It seems that the present simple model will continue to give predictions which may be considered adequate for the time being.

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APPENDIX

Numerical Solution of the Radiation Equations

The general Pun–Spalding procedure for the conservation equations is an iterative one in which values of all the flow parameters are obtained at the nodes of a rectangular calculation grid covering the flow field. At each step in this iteration process, values for (qr) and H can be obtained explicitly in terms of the other variables. Clearly, in a different situation, when the state of the radiating medium is known everywhere, the radiation fluxes may be calculated straightaway.

The point of departure is to write equations (21) and (22) in their simplest finite-difference form, thus:

$$(qr)_i = (qr)_{i-1} + (r_i - r_{i-1})(ra)_{im}[(4\sigma T^4)_{im} - 3H_{i-1}] \quad (A.1)$$

$$H_i = H_{i-1} - (r_i - r_{i-1})\left(\frac{a+s}{r}\right)_{im}(qr)_{i-1} \quad (A.2)$$

where subscripts i, $i-1$ refer to adjacent grid nodes in the $\theta - z$ plane $(r_i > r_{i-1})$ and the subscript im denotes a mean value between the two.

These equations are conveniently written in the matrix form:

$$\begin{bmatrix} qr \\ H \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & , -3ar(r_i - r_{i-1}), & 4ar\sigma T^4(r_i - r_{i-1}) \\ -\frac{r_i - r_{i-1}}{r}(a+s), & 1, & 0 \\ 0, & 0, & 1 \end{bmatrix}_{im} \begin{bmatrix} qr \\ H \\ 1 \end{bmatrix}_{i-1} \quad (A.3)$$

or, for brevity;

$$\boldsymbol{Z}_i = [A_i]\boldsymbol{Z}_{i-1} \quad (A.4)$$

where:

$\boldsymbol{Z}$ = the vector $\begin{bmatrix} qr \\ H \\ 1 \end{bmatrix}$

$[A_i]$ = a matrix operator which is a function of properties averaged between grid points $i-1$ and i.

The relationship between values of $\boldsymbol{Z}$ on the centreline and at the wall of the combustion chamber is derived as:

$$\boldsymbol{Z}_W = [A_W][A_{W-1}][A_{W-2}]\ldots[A_1]\boldsymbol{Z}_0 = [B]\boldsymbol{Z}_0 \quad (A.5)$$

where the subscripts W and 0 refer to wall and centreline values respectively. Equation (A.5) can be expanded to yield:

$$(qr)_W = \alpha_1(qr)_0 + \beta_1 H_0 + \gamma_1 \quad (A.6)$$

$$H_W = \alpha_2(qr)_0 + \beta_2 H_0 + \gamma_2 \quad (A.7)$$

where the coefficients α, β, γ are those of the matrix $[B]$. The boundary conditions are given by equations (23) and (25) which, when combined with equations (A.6) and (A.7), yield the condition:

$$H_0 = \frac{\sigma T_W{}^4 - 0{\cdot}75\,\gamma_2 + \gamma_1\left(\frac{1}{\varepsilon_W} - \frac{1}{2}\right)\frac{1}{r_W}}{0{\cdot}75\,\beta_2 - \beta_1\left(\frac{1}{\varepsilon_W} - \frac{1}{2}\right)\frac{1}{r_W}}. \quad (A.8)$$

Values of qr and H at each grid node may now be obtained from equations (A.1) and (A.2) using the centreline values obtained from equations (23) and (A.8).

UN MODÈLE SIMPLE DE TRANSFERT THERMIQUE PAR RAYONNEMENT À PARTIR D'UNE IMAGE DE PARTICULES EN COMBUSTION DANS UN COURANT GAZEUX CONFINÉ

Résumé— On décrit un modèle simple de rayonnement thermique, basé sur l'approximation de Milne–Eddington, qui a été utilisé avec un certain degré de succès pour prédire les coefficients de transfert thermique dans une flamme à symètrie axiale de combustible pulvérise

EIN EINFACHES MODELL DES WÄRMETRANSPORTES DURCH STRAHLUNG AUS EINER WOLKE VON BRENNENDEN TEILCHEN IN EINEM BEGRENZTEN GAS-STROM

Zusammenfassung—Die Arbeit beschreibt ein einfaches Modell der thermischen Strahlung, das auf der Milne–Eddington-Näherung basiert. Dieses Modell ist mit einigem Erfolg dazu verwendet worden, den Wärmestrom in einer axialsymmetrischen Pulverbrennstofflamme vorauszusagen.

ПРОСТАЯ МОДЕЛЬ ЛУЧИСТОГО ПЕРЕНОСА ТЕПЛА ОТ ОБЛАКА ГОРЯЩИХ ЧАСТИЦ В ЗАКРЫТОЙ СТРУЕ ГАЗА

Аннотация—В статье описывается простая модель теплового излучения, основанная на аппроксимации Милна-Эддингтона, которая с успехом применялась для расчета скоростей переноса тепла в осесимметричном пламени распыленного топлива.